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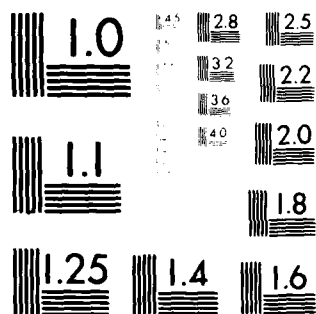
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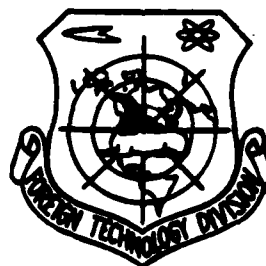


THE CHARACTERIZATION IN THE FREQUENCY DOMAIN OF THE CONTINUOUS  
LINEAR SYSTEMS, INVARIABLE IN TIME, WITH THE INDEX  
FUNCTIONS OF EXPONENTIAL-POLYNOMIAL TYPE

by

V. Cirtoaje

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## STUDIES

The characterization in the frequency domain of the continuous linear systems, invariable in time, with the index functions of exponential-polynomial type

(Caracterizarea în domeniul frecvenței a sistemelor liniare continue, invariante în timp, cu funcții indiciale de tip exponential-polinomial)

by V. CÎRTOAJE

The Institute of Petroleum and Gas, Ploiești

The paper substantiates independently, but nevertheless suggests the probability of applying the operational calculus (Fourier transform), the correlations present between the frequency domain and the time domain in a certain group of continuous linear systems and invariable in time.

Due to the polynomial composition, the group of functions of the exponential-polynomial type considered in the present paper does not accept Fourier transform. Nevertheless, the characterization in the frequency domain of the systems with index functions of the exponential-polynomial type could be made through an adequate definition of the frequency functions, based on the response in the permanent conditions of the system at the sinusoidal entry.

It is presented the methodology of determining experimentally the frequency functions, their main properties, the relations of obtaining the index function from the frequency functions, and an important theorem which expresses the necessary and sufficient conditions which have to be satisfied by the frequency functions in order to correspond to some systems with an index function of exponential-polynomial type.

In conclusion, systems in series connection and with reaction are presented.

### 1. Relations of convolution entry-exit

For a linear system with an entry  $x$ , exit  $y$  and index function  $a(t)$  integrable (in Riemann sense) on  $[0, \infty)$ , situated up to the moment  $t=0$  in a stationary regime, characterized by  $x(t) = y(t) = 0$ , for  $t < 0$ , we can calculate the exit signal  $y(t)$  for any entry signal  $x(t)$ , differentiable on  $(0, t]$ , with  $x(0) = 1/2 x(0+)$  and with the differentiable  $x'(t)$  integrable on  $[0, t]$ , with the relation\*)

$$y(t) = \begin{cases} a(0)x(0+), & t = 0, \\ a(t)x(0+) + \int_0^t a(t-\theta)x'(\theta)d\theta, & t > 0. \end{cases} \quad (1)$$

Starting from the definition of the index function, as a response to the system present in initial null conditions, when at the moment  $t=0$  it is applied at the entry a signal of unitary type:

$$H(t) = \begin{cases} 0, & t < 0, \\ 0.5, & t = 0, \\ 1, & t > 0, \end{cases}$$

the relation (1) is easily deduced based on the properties of superposition of the linear systems (fig. 1):

$$y(t) = a(t)x(0+) + \lim_{\substack{n \rightarrow \infty \\ n\Delta = t}} \sum_{k=1}^n a(t-k\Delta)[x(k\Delta) - x((k-1)\Delta)].$$



Fig. 1. - The entry signal approximated by step type signals, spaced in time.

By changing the integral variable, relation (1) is brought to the form:

$$y(t) = a(t)x(0+) + \int_0^t a(\theta)x'(t-\theta)d\theta, \quad t > 0. \quad (1')$$

\*) (from page 1) In the whole paper all the integrals will be taken to accept the following:

$$\int_a^b f(x)dx = \lim_{\substack{n \rightarrow \infty \\ a = a_0, b = a_n}} \sum_{k=0}^{n-1} f(x_k)\Delta x = \int_{a_0}^{a_n} f(x)dx, \quad a < b.$$

It can be easily proven that when  $x(t)$  is not differentiable on  $(0, t]$ , but there is a division of the interval  $[0, t]$ :

$$0 = \theta_0 < \theta_1 < \dots < \theta_k = t,$$

so that  $x(t)$  is differentiable on each interval  $(\theta_{k-1}, \theta_k)$ , to have the differential  $x'(t)$  integrable on  $[\theta_{k-1}, \theta_k]$ , and each point have finite limits and a value equal to the semi-sum of the lateral limits, the exit value  $y(t)$  is given by the relation

$$y(t) = \sum_{k=0}^i [x(\theta_k +) - x(\theta_k -)] a(t - \theta_k) + \int_0^t a(t - \theta) x'(\theta) d\theta, \quad t > 0, \quad (1'')$$

where

$$\int_0^t a(t - \theta) x'(\theta) d\theta = \sum_{k=1}^i \int_{\theta_{k-1}}^{\theta_k} a(t - \theta) x'(\theta) d\theta.$$

If, furthermore,  $a(t)$  is differentiable on  $(0, t]$  and has the differential  $a'(t)$  integrable on  $[0, t]$ , we have

$$y(t) = a(0) [x(t +) - x(t -)] + a(0 +) x(t -) + \int_0^t a'(t - \theta) x(\theta) d\theta, \quad t > 0. \quad (1''')$$

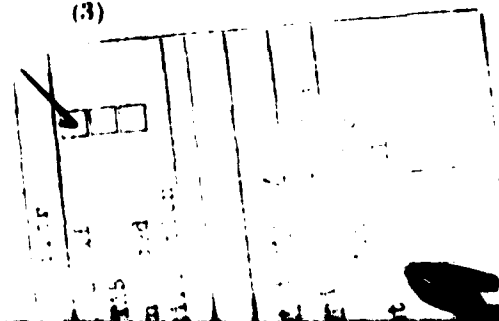
## 2. The group systems with index functions of exponential-polynomial type

This group, called from now on the group of the systems of EP type, is characterized by the property of decomposability of each system A with an index function  $a(t)$ , in two sub-systems B and C connected in parallel (fig. 2), the first with an index function  $b(t)$  of polynomial type:

$$b(t) = \begin{cases} 0,5 \, b_0 & , \, t = 0, \\ \sum_{i=0}^n b_{n-i} t^{n-i}, & t > 0, \end{cases} \quad (2)$$

and the second with an index function  $c(t)$  of "exponential" type,

$$\lim_{t \rightarrow \infty} t^p e^{pt} (t) = 0 \quad (p = 0, 1, 2, \dots), \quad (3)$$



differentiable an unlimited number of times on each interval  $(t_{k-1}, t_k)$ , where  $0=t_0 < t_1 < \dots < t_n = \infty$  and having in each point  $t_k$  a value equal to the semi-sum of the lateral limits, and its lateral limits and those of the differential finite.

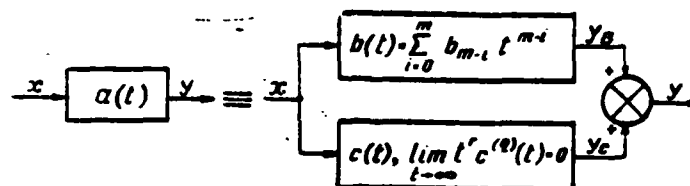


Fig. 2. - Decomposability of an EP type system according to the index function

The operation of decomposability can be easily effectuated, based on the relations presented below, in the order of their writing:

$$h_m = \lim_{t \rightarrow \infty} \frac{a(t)}{t^m},$$

$$h_{m-i} = \lim_{t \rightarrow \infty} \frac{1}{t^{m-i}} \left[ a(t) - \sum_{i=0}^{i-1} h_{m-i} t^{m-i} \right], \quad i = 1, 2, \dots, m. \quad (1)$$

$$b(t) = \sum_{i=0}^m h_{m-i} t^{m-i},$$

$$c(t) = a(t) - b(t).$$

### 3. The determination of the frequency functions from the index function

Let us assume that at the entry of the system A we apply a signal which contains the sinusoidal component:

$$x(t) = A \sin(\omega t + \varphi).$$

With the help of the relation (1') we can find the answers of the sub-systems B and C corresponding to this entry sinusoidal component:

$$y_B(t) = -A \sum_{i=0}^{m-1} (m-i)! b_{m-i} \sum_{k=1}^{m-i} \frac{(-1)^k t^{m-i-k}}{(m-i-k)! \omega^k} \sin\left(\varphi + \frac{k\pi}{2}\right) +$$

$$+ A \sin(\omega t + \varphi) \sum_{i=0}^m \frac{(m-i)! b_{m-i}}{\omega^{m-i}} \cos \frac{(m-i)\pi}{2} -$$

$$- A \cos(\omega t + \varphi) \sum_{i=0}^m \frac{(m-i)! b_{m-i}}{\omega^{m-i}} \sin \frac{(m-i)\pi}{2}.$$



$$\begin{aligned}
y_e(t) &= A \sin \varphi c(t) + A \omega \int_0^t c(\theta) \cos [\omega(t - \theta) + \varphi] d\theta = \\
&= A \sin \varphi c(t) + A \omega \sin(\omega t + \varphi) \int_0^t c(\theta) \sin \omega \theta d\theta + \\
&\quad + A \omega \cos(\omega t + \varphi) \int_0^t c(\theta) \cos \omega \theta d\theta.
\end{aligned}$$

Taking into consideration the relation (3), the limits below,

$$\lim_{t \rightarrow \infty} \int_0^t c(\theta) \sin \omega \theta d\theta = \int_0^\infty c(\theta) \sin \omega \theta d\theta,$$

$$\lim_{t \rightarrow \infty} \int_0^t c(\theta) \cos \omega \theta d\theta = \int_0^\infty c(\theta) \cos \omega \theta d\theta,$$

exist and are finite. In consequence, in a permanent regime, the exit signal of the system contains the sinusoidal component:

$$\begin{aligned}
y(t) &= A \left[ \sum_{i=0}^n \frac{(m-i)! b_{m-i}}{\omega^{m-i}} \cos \frac{(m-i)\pi}{2} + \omega \int_0^\infty c(\theta) \sin \omega \theta d\theta \right] \sin(\omega t + \varphi) \\
&\quad + A \left[ - \sum_{i=0}^n \frac{(m-i)! b_{m-i}}{\omega^{m-i}} \sin \frac{(m-i)\pi}{2} + \omega \int_0^\infty c(\theta) \cos \omega \theta d\theta \right] \cos(\omega t + \varphi),
\end{aligned}$$

based on which there are defined the real frequency functions:

$$U(\omega) = \sum_{i=0}^n \frac{(m-i)! b_{m-i}}{\omega^{m-i}} \cos \frac{(m-i)\pi}{2} + \omega \int_0^\infty c(\theta) \sin \omega \theta d\theta, \quad (5)$$

$$V(\omega) = - \sum_{i=0}^n \frac{(m-i)! b_{m-i}}{\omega^{m-i}} \sin \frac{(m-i)\pi}{2} + \omega \int_0^\infty c(\theta) \cos \omega \theta d\theta, \quad (6)$$

and the complex frequency function:

$$F(j\omega) = U(\omega) + jV(\omega) = \sum_{i=0}^n \frac{(m-i)! b_{m-i}}{(j\omega)^{m-i}} + j\omega \int_0^\infty c(\theta) e^{-j\omega\theta} d\theta. \quad (7)$$

From the expression of the sinusoidal component of the exit signal  $y_p(t)$  in a permanent regime, we obtain the frequency functions of the sub-system B:

$$U_s(\omega) = \sum_{i=0}^m \frac{(m-i)! b_{m-i}}{\omega^{m-i}} \cos \frac{(m-i)\pi}{2}, \quad (8)$$

$$V_s(\omega) = - \sum_{i=0}^m \frac{(m-i)! b_{m-i}}{\omega^{m-i}} \sin \frac{(m-i)\pi}{2}, \quad (9)$$

$$F_s(j\omega) = \sum_{i=0}^m \frac{(m-i)! b_{m-i}}{(j\omega)^{m-i}}, \quad (10)$$

and from the expression of the sinusoidal component of the exit signal  $y_c(t)$  in a permanent regime, we obtain the frequency functions of the sub-system C:

$$U_c(\omega) = \omega \int_0^{\infty} c(\theta) \sin \omega \theta d\theta, \quad (11)$$

$$V_c(\omega) = \omega \int_0^{\infty} c(\theta) \cos \omega \theta d\theta, \quad (12)$$

$$F_c(j\omega) = j\omega \int_0^{\infty} c(\theta) e^{-j\omega\theta} d\theta. \quad (13)$$

#### Observations

a) Starting from the index function of the system A:

$$a(t) = \sum_{i=0}^m b_{m-i} t^{m-i} + c(t),$$

we find for the transfer function the following expression:

$$F(s) = \sum_{i=0}^m \frac{(m-i)! b_{m-i}}{s^{m-i}} + s \int_0^{\infty} c(t) e^{-st} dt.$$

The relation (7) proves that in the EP type system the complex frequency function can be obtained directly from the transfer function by replacing the complex variable  $s$  with an imaginary variable  $j\omega$ .

b) The complex frequency function  $F(j\omega)$  of an EP type system can be well defined, as being the ratio between the representation in the complex  $Y(t)$  of the sinusoidal component  $y(t)$  at the exit of the system (in a permanent regime) and the representation in the complex  $X(t)$  of the sinusoidal component  $x(t)$  from the entry. True,

$$X(t) = A e^{j(\omega t + \varphi)},$$

$$Y(t) = A U(\omega) e^{j(\omega t + \varphi)} + A V(\omega) e^{j(\omega t + \varphi + \frac{\pi}{2})} = A e^{j(\omega t + \varphi)} [U(\omega) + jV(\omega)]$$

and thus

$$\frac{Y(t)}{X(t)} = U(\omega) + jV(\omega) = F(j\omega). \quad (14)$$

#### 4. Decomposability of the EP type systems after the frequency functions

Taking into consideration the relations (7), (10) and (13), the decomposition of a system A with a complex frequency function  $F(j\omega)$  in the two sub-systems B and C, takes place with the help of the relations:

$$b_m = \frac{1}{m!} \lim_{\omega \rightarrow 0} (j\omega)^m F(j\omega),$$

$$b_{m-i} = \frac{1}{(m-i)!} \lim_{\omega \rightarrow 0} \left[ (j\omega)^{m-i} F(j\omega) - \sum_{l=0}^{i-1} \frac{(m-l)! b_{m-l}}{(j\omega)^{l-i}} \right], i = 1, 2, \dots, m,$$

$$F_B(j\omega) = \sum_{i=0}^m \frac{(m-i)! b_{m-i}}{(j\omega)^{m-i}}, \quad (15)$$

$$F_C(j\omega) = F(j\omega) - F_B(j\omega).$$

Also, taking into consideration (5), (6), (8), (9), (11) and (12), the decomposition relations after the real frequency functions  $U(\omega)$  and  $V(\omega)$ , are the following:

$$b_m = \begin{cases} \frac{(-1)^{\frac{m}{2}}}{m!} \lim_{\omega \rightarrow 0} \omega^m U(\omega), & m = \text{even} \\ \frac{(-1)^{\frac{m+1}{2}}}{m!} \lim_{\omega \rightarrow 0} \omega^m V(\omega), & m = \text{odd} \end{cases}$$

$$b_{m-i} = \begin{cases} \frac{(-1)^{\frac{m-i}{2}}}{(m-i)!} \lim_{\omega \rightarrow 0} \left[ \omega^{m-i} U(\omega) - \sum_{l=0}^{i-1} \frac{(m-l)! b_{m-l}}{\omega^{l-i}} \cos \frac{(m-l)\pi}{2} \right], & m-i = \text{even} \\ \frac{(-1)^{\frac{m-i+1}{2}}}{(m-i)!} \lim_{\omega \rightarrow 0} \left[ \omega^{m-i} V(\omega) - \sum_{l=0}^{i-1} \frac{(m-l)! b_{m-l}}{\omega^{l-i}} \sin \frac{(m-l)\pi}{2} \right], & m-i = \text{odd} \end{cases} \quad (16)$$

$$U_B(\omega) = \sum_{i=0}^m \frac{(m-i)! b_{m-i}}{\omega^{m-i}} \cos \frac{(m-i)\pi}{2}.$$

$$V_B(\omega) = - \sum_{i=0}^m \frac{(m-i)! b_{m-i}}{\omega^{m-i}} \sin \frac{(m-i)\pi}{2}.$$

$$U_C(\omega) = U(\omega) - U_B(\omega),$$

$$V_C(\omega) = V(\omega) - V_B(\omega).$$

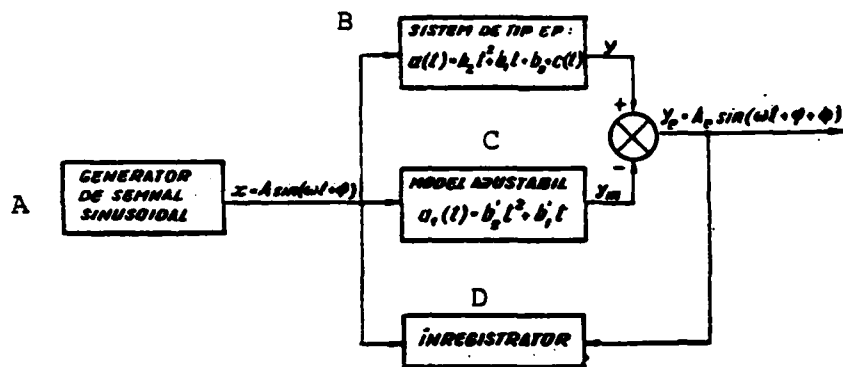
## 5. The experimental determination of the frequency functions

Starting from the fact that the frequency functions of the exponential-parabolic systems are defined by the expression:

$$y(t) = A \left( \frac{2h_2}{\omega} t + \frac{h_1}{\omega} \right) + A U(\omega) \sin(\omega t + \varphi) + A V(\omega) \cos(\omega t + \varphi),$$

which expresses the value in a permanent regime, of the exit signal in the system, when at the entry there is applied a sinusoidal signal  $x = A \sin(\omega t + \varphi)$ , we easily grasp the methodology of determining experimentally the frequency functions of the systems of this type (fig. 3).

We adjust  $b'_1$  until the exit signal  $y_e$  oscillates around a constant value, and then  $b'_1$  is adjusted until this constant value becomes zero. The values found for the coefficients  $b'_1$  and  $b'_2$  are independent of the frequency of the entry signal.



Translation Note: A - Generator of sinusoidal signal  
 B - EP type system  
 C - Adjustable model  
 D - Recorder

Fig. 3. - Diagram of experimental determination of the frequency function in the systems with exponential-parabolic index function ( $m=2$ )

The frequency functions are determined for the various values of  $\omega$ , with the aid of the relations:

$$\dot{U}(\omega) = -\frac{b'_2}{\omega^2} + \frac{A_s(\omega)}{A} \cos \Phi(\omega), \quad (16)$$

$$V(\omega) = \frac{b'_1}{\omega} + \frac{A_s(\omega)}{A} \sin \Phi(\omega), \quad (17)$$

$$F(j\omega) = \frac{b'_2}{(j\omega)^2} + \frac{b'_1}{j\omega} + \frac{A_s(\omega)}{A} e^{j\Phi(\omega)}. \quad (18)$$

In the case when the component  $b(t)$  of the index function is represented by a polynomial of a higher grade,  $m > 2$ , the coefficients of this polynomial can be determined experimentally with the aid of a diagram based on the relations of decomposibility.

## 6. Some properties of the frequency functions

a) From the relations (5) and (6) we find the frequency function is not influenced by the components  $b_i t^i$ ,  $i$  - odd, of the index function, and the frequency function  $V(\omega)$  is not influenced by the components  $b_i t^i$ ,  $i$  - even, of the same function. Thus,  $U(\omega)$  does not contain the information regarding the simple integral character, triple integral, etc., of the system, and  $V(\omega)$  does not contain information regarding the proportional character, double integral, etc., of the system. Hereby we have the conclusion that the frequency functions  $U(\omega)$  and  $V(\omega)$ , taken separately do not ensure the complete characterization of an EP type system (of its index function). In the following paragraph we will see that both  $U(\omega)$  and  $V(\omega)$  permit the complete determination of the index function  $c(t)$ , and the complex frequency function  $F(j\omega)$ , through its two components  $U(\omega)$  and  $V(\omega)$ , it enables the complete determination of the index function  $a(t)$  in the group of EP type systems.

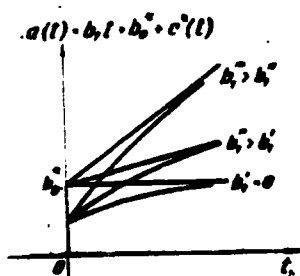


Fig. 4. - The group of index exponential-linear functions, characterized by the same frequency function  $U(\omega)$ .

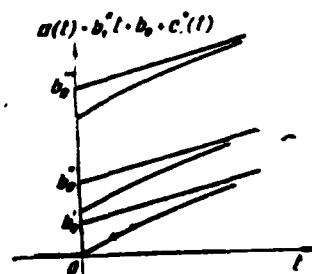


Fig. 5. - The group of index exponential-linear functions characterized by the same frequency function  $V(\omega)$ .

b) Based on the relations (3), (10) and (13), we find that

the frequency functions are differentiable an unlimited number of times in relation to  $\omega$  on the interval  $(0, \infty)$  [2], and

$$\frac{d^r F_c(j\omega)}{d\omega^r} = \int_0^\infty (-r + j\omega\theta) \theta^{r-1} c(\theta) e^{-j(\omega\theta + \frac{\pi}{2})} d\theta, \quad r \geq 0, \quad (19)$$

from here results

$$\lim_{\omega \rightarrow 0} \frac{d^r F_c(j\omega)}{d\omega^r} = -r e^{-j\frac{\pi}{2}} \int_0^\infty \theta^{r-1} c(\theta) d\theta, \quad (20)$$

and from here

$$\lim_{\omega \rightarrow 0} \frac{d^r U_c(\omega)}{d\omega^r} = \begin{cases} (-1)^{\frac{r+1}{2}} r \int_0^\infty \theta^{r-1} c(\theta) d\theta, & r = \text{even} \\ 0 & , r = \text{odd} \end{cases} \quad (21)$$

$$\lim_{\omega \rightarrow 0} \frac{d^r V_c(\omega)}{d\omega^r} = \begin{cases} 0 & , r = \text{even} \\ (-1)^{\frac{r-1}{2}} r \int_0^\infty \theta^{r-1} c(\theta) d\theta, & r = \text{odd} \end{cases} \quad (22)$$

Also from the relation (13) we obtain

$$\frac{d^r}{d\omega^r} \left[ \frac{F_c(j\omega)}{\omega} \right] = \int_0^\infty \theta^r c(\theta) e^{-j[\omega\theta + \frac{(r-1)\pi}{2}]} d\theta, \quad r \geq 0, \quad (23)$$

from where it results

$$\lim_{\omega \rightarrow 0} \frac{d^r}{d\omega^r} \left[ \frac{F_c(j\omega)}{\omega} \right] = e^{-j\frac{(r-1)\pi}{2}} \int_0^\infty \theta^r c(\theta) d\theta, \quad (24)$$

and from here

$$\lim_{\omega \rightarrow 0} \frac{d^r}{d\omega^r} \left[ \frac{U_c(\omega)}{\omega} \right] = \begin{cases} 0 & , r = \text{even}, \\ (-1)^{\frac{r-1}{2}} \int_0^\infty \theta^r c(\theta) d\theta, & r = \text{odd}, \end{cases} \quad (25)$$

$$\lim_{\omega \rightarrow 0} \frac{d^r}{d\omega^r} \left[ \frac{V_c(\omega)}{\omega} \right] = \begin{cases} (-1)^{\frac{r}{2}} \int_0^\infty \theta^r a(\theta) d\theta, & r = \text{even}, \\ 0, & r = \text{odd}, \end{cases} \quad (26)$$

c) Following the integration in parts, the relation (7) becomes:

$$F(j\omega) = \sum_{i=0}^m \frac{(m-i)! b_{m-i}}{(j\omega)^{m-i}} + \sum_{k=0}^{n-1} [c(t_{k+}) - c(t_{k-})] e^{-j\omega t_k} + \sum_{k=1}^n \int_{t_{k-1}}^{t_k} c'(\theta) e^{-j\omega \theta} d\theta, \quad (27)$$

from where, by repeated differentiations, we obtain

$$\begin{aligned} \frac{d^r F(j\omega)}{d\omega^r} &= \sum_{i=0}^m \frac{(m-i+r-1)! (m-i) b_{m-i}}{(j\omega)^{m-i+r}} e^{-j\frac{r\pi}{2}} + \\ &+ \sum_{k=0}^{n-1} [c(t_{k+}) - c(t_{k-})] t_k^r e^{-j(\omega t_k + \frac{r\pi}{2})} + \\ &+ \sum_{k=1}^n \int_{t_{k-1}}^{t_k} \theta^r c'(\theta) e^{-j(\omega \theta + \frac{r\pi}{2})} d\theta, \quad r \geq 1. \end{aligned} \quad (28)$$

If the index function  $c(t)$  is continuous on  $(0, \infty)$ , direct [2] or by the effectuation of a new integration in parts, from the relations (27) and (28) it results

$$\lim_{\omega \rightarrow \infty} F(j\omega) = h_0 + c(0+) = a(0+), \quad (29)$$

$$\lim_{\omega \rightarrow \infty} \frac{d^r F(j\omega)}{d\omega^r} = 0, \quad r \geq 1. \quad (30)$$

Following the successive integration in parts, from the relation (13) results

$$F_c(j\omega) = \sum_{i=0}^{r-1} F_{ci}^*(j\omega) + F_{cr}(j\omega), \quad (31)$$



where

$$F_{c_l}^*(j\omega) = \frac{1}{(j\omega)^l} \sum_{k=0}^{q-1} [c^{(l)}(t_k+) - c^{(l)}(t_k-)] e^{-j\omega t_k}, \quad l = 0, 1, \dots, q-1, \quad (32)$$

$$F_{c_q}(j\omega) = \frac{1}{(j\omega)^{q-1}} \sum_{k=1}^q \int_{t_{k-1}}^{t_k} c^{(q)}(\theta) e^{-j\omega\theta} d\theta. \quad (33)$$

Since

$$\begin{aligned} & \omega^q \frac{d^r}{d\omega^r} \left[ \frac{F_{c_q}(j\omega)}{\omega} \right] = \\ & = \sum_{s=0}^r \frac{(-1)^{r-s} (q+r-s)! q! c_q^{(s)}}{q! (q+r-s)\omega^{r-s}} \sum_{k=1}^q \int_{t_{k-1}}^{t_k} \theta^s c^{(q)}(\theta) e^{-j[\omega\theta + (q+s-1)\frac{\pi}{2}]} d\theta, \quad (34) \end{aligned}$$

direct [2], or by a new integration in parts, we obtain

$$\lim_{\omega \rightarrow \infty} \omega^q \frac{d^r}{d\omega^r} \left[ \frac{F_{c_q}(j\omega)}{\omega} \right] = 0, \quad q > 0, \quad r \geq 0. \quad (35)$$

d) From the relations (5) and (6), we find immediately that the frequency function  $U(\omega)$  is even, and the frequency function  $V(\omega)$  is odd:

$$U(\omega) = U(-\omega), \quad (36)$$

$$V(\omega) = -V(-\omega). \quad (37)$$

e) If the index functions of the two systems  $A_1$  and  $A_2$  are connected by the relation

$$a_1(t) = a_2(\alpha t), \quad (38)$$

then, from (7), it results that the complex frequency functions of the systems satisfy the relation

$$F_1(j\omega) = F_2 \left( j \frac{\omega}{\alpha} \right). \quad (39)$$

In consequence, at an "expansion" ("contraction") of the index function, there is a "contraction" ("expansion") of the frequency function.

## 7. Determination of the index function from the frequency functions

The index function  $c(t)$  can be represented on the interval  $(0, \infty)$  by the two real Fourier integrals:

$$\begin{aligned} c(t) &= \frac{2}{\pi} \int_0^{\infty} \sin \omega t d\omega \int_0^{\infty} c(\theta) \sin \omega \theta d\theta = \\ &= \frac{2}{\pi} \int_0^{\infty} \cos \omega t d\omega \int_0^{\infty} c(\theta) \cos \omega \theta d\theta, \end{aligned} \quad (40)$$

from where, based on the relations (11) and (12) it results

$$c(t) = \frac{2}{\pi} \int_0^{\infty} \frac{U_c(\omega)}{\omega} \sin \omega t d\omega, \quad t > 0, \quad (41)$$

$$c(t) = \frac{2}{\pi} \int_0^{\infty} \frac{V_c(\omega)}{\omega} \cos \omega t d\omega, \quad t > 0, \quad (42)$$

and from here

$$a(t) = \sum_{i=0}^m b_{m-i} t^{m-i} + \frac{2}{\pi} \int_0^{\infty} \frac{U_c(\omega)}{\omega} \sin \omega t d\omega, \quad t > 0, \quad (43)$$

$$a(t) = \sum_{i=0}^m b_{m-i} t^{m-i} + \frac{2}{\pi} \int_0^{\infty} \frac{V_c(\omega)}{\omega} \cos \omega t d\omega, \quad t > 0. \quad (44)$$

Taking into consideration that  $U_c(\omega)$  is even and  $V_c(\omega)$  is odd, the relations (41) and (42) permit the expression of the component  $c(t)$  of the index function in relation to the complex frequency function  $F_c(j)$ , in the following two forms:

$$c(t) = \frac{1}{\pi} \operatorname{Re} \int_0^{\infty} \frac{F_c(j\omega)}{j\omega} e^{j\omega t} d\omega = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{F_c(j\omega)}{j\omega} e^{j\omega t} d\omega. \quad (45)$$

In conformity with the relations (41) and (42),  $c(t)$  can be determined either in function of the frequency  $U_c(\omega)$  or of the  $V_c(\omega)$ .

Since  $U_c(\omega)$  can be obtained from  $U(\omega)$  and  $V_c(\omega)$ , it results that  $c(t)$  can be determined either from the frequency function  $U(\omega)$  or from  $V(\omega)$ .

From the relations (43), (44) and of the decomposability of the EP type systems after the frequency functions, it results that in order to determine the index function  $a(t)$  we must know either  $U(\omega)$  and its behaviour around the value  $\omega = 0$  of the function  $V(\omega)$  (or  $V_B(\omega)$ ), or  $V(\omega)$  and its behaviour around the value  $\omega = 0$  of the function  $U(\omega)$  (or  $U_B(\omega)$ ).

#### 8. Parseval theorem

Using in turn the relations (41) and (11), we have

$$\begin{aligned} \int_0^\infty c^2(t) dt &= \frac{2}{\pi} \int_0^\infty c(t) dt \int_0^\infty \frac{U_c(\omega)}{\omega} \sin \omega t d\omega = \\ &= \frac{2}{\pi} \int_0^\infty \frac{U_c(\omega)}{\omega} d\omega \int_0^\infty c(t) \sin \omega t dt = \frac{2}{\pi} \int_0^\infty \left[ \frac{U_c(\omega)}{\omega} \right]^2 d\omega. \end{aligned}$$

Similarly, based on the relations (42) and (12) we obtain

$$\int_0^\infty c^2(t) dt = \frac{2}{\pi} \int_0^\infty \left[ \frac{V_c(\omega)}{\omega} \right]^2 d\omega,$$

and from here

$$\begin{aligned} \int_0^\infty c^2(t) dt &= \frac{2}{\pi} \int_0^\infty \left[ \frac{U_c(\omega)}{\omega} \right]^2 d\omega = \\ &= \frac{2}{\pi} \int_0^\infty \left[ \frac{V_c(\omega)}{\omega} \right]^2 d\omega = \frac{1}{\pi} \int_0^\infty \left| \frac{F_c(j\omega)}{j\omega} \right|^2 d\omega. \end{aligned} \quad (46)$$

#### 9. Integral correlations between the real frequency functions

From (11) and (41), respectively (12) and (42), we obtain the following relations of dependence between  $U_c(\omega)$  and  $V_c(\omega)$ :

$$\frac{U_c(\omega)}{\omega} = \frac{2}{\pi} \int_0^\infty \sin \omega t dt \int_0^\infty \frac{V_c(\omega)}{\omega} \cos \omega t d\omega, \quad (47)$$

$$\frac{U_c(\omega)}{\omega} = \frac{2}{\pi} \int_0^{\infty} \cos \omega t dt \int_0^{\infty} \frac{U_c(\omega)}{\omega} \sin \omega t d\omega. \quad (18)$$

10. The recognition of some EP type systems according to the frequency functions

From the properties of the frequency functions deduced from the paragraph 6, there are obvious the following conditions necessary for a real frequency function  $U(\omega)$  to characterize an EP type system:

a) The values of the coefficients  $b_{m-i}(m-i=0, 2, 4, \dots)$ , resultant from the relations of decomposibility of  $U(\omega)$ , to be finite;

b)  $U(\omega)$  must be an unlimited number of times differentiable on  $(0, \infty)$ ;

$$c) \lim_{\omega \rightarrow 0} \frac{d^r}{d\omega^r} \left[ \frac{U_c(\omega)}{\omega} \right] = \begin{cases} 0, & r = 0, 2, 4, \dots \\ \text{finite}, & r = 1, 3, 5, \dots \end{cases}$$

where

$$U_c(\omega) = U(\omega) - \sum_{i=0}^m \frac{(m-i)! b_{m-i}}{\omega^{m-i}} \cos \frac{(m-i)\pi}{2};$$

d) for any  $q \geq 1$ :

$$U_c(\omega) = \sum_{l=0}^{q-1} U_{cl}^*(\omega) + U_{cq}(\omega),$$

where

$$U_{cl}^*(\omega) = \frac{1}{\omega^l} \sum_{k=0}^{n-1} x_{lk} \cos \left( \omega t_k + \frac{l\pi}{2} \right), \quad 0 = t_0 < t_1 < \dots < t_{n-1}$$

and

$$\lim_{\omega \rightarrow \infty} \omega^r \frac{d^r}{d\omega^r} \left[ \frac{U_{cq}(\omega)}{\omega} \right] = 0, \quad (\forall) r \geq 0.$$

Further on we will show that these conditions are also sufficient for  $U(\omega)$  to be the real frequency function of an EP type system, that is to exist an index function  $a(t)$  of the polynomial-exponential type, to which it corresponds the real frequency function  $U(\omega)$ .

For this we will prove that:

- the function

$$c(t) = \frac{2}{\pi} \int_0^{\infty} \frac{U_c(\omega)}{\omega} \sin \omega t d\omega, \quad t > 0,$$

satisfies the relations

$$\lim_{t \rightarrow \infty} t^p c^{(p)}(t) = 0, \quad p = 0, 1, 2, \dots$$

is differentiable an unlimited number of times on the interval  $(t_{k-1}, t_k)$ , where  $k=1, 2, \dots, n-1$  and  $t_n = \infty$ , has in every point  $t_k$  a value equal to the semi-sum of its lateral limits and its lateral limits and those of its differentials are finite;

- the system with the index function:

$$a(t) = \sum_{m=0}^{\infty} b_m t^m + c(t)$$

has  $U(\omega)$  as a real frequency function.

Performing successive integrations in parts and taking into consideration the properties b), c), and d), the function  $c(t)$  is brought to the form:

$$c(t) = \frac{2}{\pi^{r+1}} \int_0^{\infty} \left[ \frac{U_c(\omega)}{\omega} \right]^{(r+1)} \sin \left[ \omega t + \frac{(r+1)\pi}{2} \right] d\omega, \quad (V) \quad r \geq -1, \quad (49)$$

from where it results

$$\lim_{t \rightarrow \infty} t^r c(t) = 0.$$

Further the function  $c(t)$  is decomposed in two components:

$$c_1(t) = \frac{2}{\pi^{r+1}} \int_0^1 \left[ \frac{U_c(\omega)}{\omega} \right]^{(r+1)} \sin \left[ \omega t + \frac{(r+1)\pi}{2} \right] d\omega, \quad (50)$$

$$c_2(t) = \frac{2}{\pi^{r+1}} \int_1^\infty \left[ \frac{U_C(\omega)}{\omega} \right]^{r+1} \sin \left[ \omega t + \frac{(r+1)\pi}{2} \right] d\omega. \quad (51)$$

For  $c_1(t)$  we find

$$c_1^{(p)}(t) = \frac{2}{\pi} \sum_{i=0}^p \frac{(-1)^{p-i} C_p(r+p-i)!}{r! p^{r+p-i+1}} \int_0^1 \omega^i \left[ \frac{U_C(\omega)}{\omega} \right]^{r+1} \sin \left[ \omega t + \frac{(r+i+1)\pi}{2} \right] d\omega \quad (52)$$

from where it results

$$\lim_{p \rightarrow \infty} p^r c_1^{(p)}(t) = 0.$$

Taking into consideration the property d), the function  $c_2(t)$  can be written as follows:

$$c_2(t) = \sum_{l=0}^{r-1} c_{2l}^*(t) + c_{2r}(t), \quad (53)$$

where

$$\begin{aligned} c_{2l}^*(t) &= \frac{2}{\pi l^{r+1}} \int_1^\infty \left[ \frac{U_{Cl}^*(\omega)}{\omega} \right]^{r+1} \sin \left[ \omega t + \frac{(r+1)\pi}{2} \right] d\omega = \\ &= \frac{2}{\pi l^{r+1}} \sum_{k=0}^{r-1} a_{lk} \sum_{i=0}^{r-1} \frac{(-1)^{r-i+1} C_{r+1}^i(l+r-i+1)! t_k}{l!} \times \\ &\times \int_1^\infty \frac{\cos \left[ \omega t_k + \frac{(l+i)\pi}{2} \right] \sin \left[ \omega t + \frac{(r+1)\pi}{2} \right]}{\omega^{l+r-i+2}} d\omega = \\ &= \frac{1}{\pi l^{r+1}} \sum_{k=0}^{r-1} a_{lk} \sum_{i=0}^{r-1} \frac{(-1)^{r-i+1} C_{r+1}^i(l+r-i+1)! t_k}{l!} \times \end{aligned}$$

$$\times \left\{ (l + l_k)^{l+r-i+1} \int_{l+l_k}^{\infty} \frac{\sin \left[ \theta + \frac{(l+r+i+1)\pi}{2} \right]}{\theta^{l+r-i+2}} d\theta + \right. \\ \left. + (l - l_k)^{l+r-i+1} \int_{l-l_k}^{\infty} \frac{\sin \left[ \theta + \frac{(-l+r-i-1)\pi}{2} \right]}{\theta^{l+r-i+2}} d\theta \right\}. \quad (54)$$

$$c_{20}^q(t) = \frac{2}{\pi^{q+1}} \int_1^{\infty} \left[ \frac{U_{Cq}(\omega)}{\omega} \right]^{q+1} \sin \left[ \omega t + \frac{(r+1)\pi}{2} \right] d\omega. \quad (55)$$

We have

$$\lim_{t \rightarrow \infty} t^r c_{21}^q(t) = 0,$$

and so much more:

$$\lim_{t \rightarrow \infty} t^r c_{22}^{q(p)}(t) = 0.$$

For  $q > p + 2$ , there is the equality

$$\lim_{t \rightarrow \infty} \omega^{p+2} \left[ \frac{U_{Cq}(\omega)}{\omega} \right]^{q+1} = 0,$$

which enables us to write the relation

$$c_{24}^{(p)}(t) = \frac{2}{\pi} \sum_{i=0}^p \frac{(-1)^{p-i} (l_p^q (r+p-i)!) }{r! (p+r-i+1)!} \int_1^{\infty} \omega^i \left[ \frac{U_{Cq}(\omega)}{\omega} \right]^{q+1} \times \\ \times \sin \left[ \omega t + \frac{(r+i+1)\pi}{2} \right] d\omega, \quad (56)$$

from where it results

$$\lim_{t \rightarrow \infty} t^r c_{24}^{(p)}(t) = 0,$$

and taking into consideration the previous results,

$$\lim_{t \rightarrow \infty} t^p c^{(p)}(t) = 0.$$

For  $r + 1 = 0$ , the function  $c_l(t)$  is an unlimited number of times differentiable on  $(0, \infty)$ ,  $c_{2l}(t)$  is differentiable on the same interval at least  $(q - 2)$  times ( $q$  - arbitrarily chosen), and

$$c_{2l}^*(t) = \frac{2}{\pi} \sum_{k=0}^n \alpha_{lk} \int_1^{\infty} \frac{\cos \left( \omega t_k + \frac{l\pi}{2} \right) \sin \omega t}{\omega^{l+1}} d\omega. \quad (57)$$

For  $l = 0$ , we obtain

$$c_{20}^*(t) = \sum_{k=0}^{n-1} \alpha_{lk} [H(t - t_k) - Si(t + t_k) - Si(t - t_k)], \quad (58)$$

where by  $H(t - t_k)$  we marked the Heaviside function:

$$H(t - t_k) = \begin{cases} 0, & t < t_k, \\ 0.5 & t = t_k \\ 1 & t > t_k. \end{cases}$$

In the points  $t_k$ ,  $k \geq 0$ , the function  $c_{20}^*(t)$  has a value equal to the semi-sum of its lateral limits, and the functions  $c_{2l}^*(t)$ ,  $l \geq 0$ , are continuous.

For each interval  $(t_{k-1}, t_k)$ , the functions  $c_{2l}^*(t)$  are differentiable an unlimited number of times, and in the points  $t_k$  the lateral limits of their differentials are finite:

$$c_{2l}^{(p)*}(t) = \begin{cases} \frac{2}{\pi} \sum_{k=0}^{n-1} \alpha_{lk} \int_1^{\infty} \frac{\cos \left( \omega t_k + \frac{l\pi}{2} \right) \sin \left( \omega t + \frac{p\pi}{2} \right)}{\omega^{l-p+1}} d\omega, & p < l, \\ \sum_{k=0}^{n-1} \alpha_{lk} \left[ \frac{(-1)^l - 1}{2} + H(t - t_k) - \right. \\ \quad \left. - \frac{(-1)^l}{\pi} Si(t + t_k) - \frac{1}{\pi} Si(t - t_k) \right] & p = l, \\ - \frac{1}{\pi} \sum_{k=0}^{n-1} \alpha_{lk} \left\{ (-1)^l \left[ \frac{\sin(t + t_k)}{t + t_k} \right]^{(p-l-1)} + \right. \\ \quad \left. + \left[ \frac{\sin(t - t_k)}{t - t_k} \right]^{(p-l-1)} \right\}, & p > l. \end{cases} \quad (59)$$



In addition,

$$c^{(p)}(t_k+) - c^{(p)}(t_k-) = c_{2p}^{*(p)}(t_k+) - c_{2p}^{*(p)}(t_k-) = x_{pk}. \quad (60)$$

At last, taking into consideration the relation (5), the system with the index function  $a(t)$  accepts the following real frequency function:

$$U^*(\omega) = \sum_{i=0}^{\infty} \frac{(m-i)! b_{m-i}}{\omega^{m-i}} \cos \frac{(m-i)\pi}{2} + \\ + \frac{2\omega}{\pi} \int_0^{\infty} \sin \omega \theta d\theta \int_0^{\infty} \frac{U_c(\omega)}{\omega} \sin \omega \theta d\omega$$

and since  $\frac{U_c(\omega)}{\omega}$  can be represented by a Fourier integral:

$$\frac{U_c(\omega)}{\omega} = \frac{2}{\pi} \int_0^{\infty} \sin \omega \theta d\theta \int_0^{\infty} \frac{U_c(\omega)}{\omega} \sin \omega \theta d\omega,$$

it results

$$U^*(\omega) = \sum_{i=0}^{\infty} \frac{(m-i)! b_{m-i}}{\omega^{m-i}} \cos \frac{(m-i)\pi}{2} + U_c(\omega) = U(\omega).$$

Similarly, we can present and demonstrate each theorem which can express the necessary and sufficient conditions in order that a real frequency function  $V(\omega)$ , respectively a complex frequency function  $F(j\omega)$  to characterize an EP type system.

#### 11. Series connection

We will show that by connecting in series two systems  $A_1$  and  $A_2$  of EP type, with complex frequency functions  $F_1(j\omega)$  and  $F_2(j\omega)$ , we obtain a system  $A$ , also of EP type, with the frequency function:

$$F(j\omega) = F_1(j\omega) \cdot F_2(j\omega). \quad (61)$$

Based on the properties of decomposability of the EP type systems, the series connection of the two systems can be represented in the form given in the figure 6.

If the regime is stationary, characterized by the null initial conditions and it is applied at  $t=0$  an entry signal  $x(t)$  of unitary step type, we obtain at the exits of the systems  $A_1$  and  $A_2$  the index functions of these systems:

$$y_a(t) = h_1(t),$$

$$y_c(t) = c_1(t),$$

$$y(t) = a(t).$$

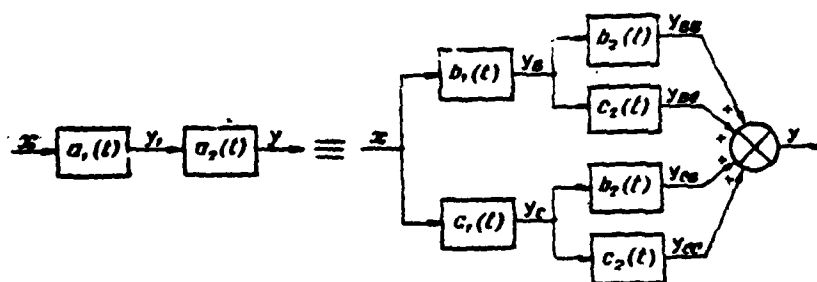


Fig. 6. - EP type systems in series connection

Attaching the index 1 to the values which characterize the system  $A_1$ , and the index 2 to the values which characterize the system  $A_2$ , based on the convolution relations, respectively (1), (1'), (1'') and (1'''), for  $t > 0$ , we have:

$$y_{BB}(t) = \sum_{i=0}^m \sum_{k=0}^{m-i} \frac{h_{1k} h_{2,m-i-k}}{c_{m-i}^k} t^{m-i}, \quad m = m_1 + m_2, \quad (62)$$

$$y_{BC}(t) = h_{10}c_2(t) + \sum_{i=0}^{m_1-1} (m_1 - i) h_{1i} \int_0^t (t - \theta)^{m_1-i-1} c_2(\theta) d\theta, \quad (63)$$

$$y_{CB}(t) = h_{20}c_1(t) + \sum_{i=0}^{m_2-1} (m_2 - i) h_{2i} \int_0^t (t - \theta)^{m_2-i-1} c_1(\theta) d\theta, \quad (64)$$

$$y_{cc}(t) = \sum_{k=0}^{n_1} [c_1(\theta_{1k}+) - c_1(\theta_{1k}-)] c_2(t - \theta_{1k}) + \sum_{k=1}^{n_1} \int_{\theta_{1,k-1}}^{\theta_{1k}} c_2(t - \theta) c'_1(\theta) d\theta, \quad (65)$$

and for  $t=0$ :

$$y(t) = a_2(0) a_1(0+). \quad (66)$$

From these relations we obtain the following expressions for the polynomial component  $b(t)$  and the "exponential" component  $c(t)$ , which make up the index function  $a(t)$  of the resultant system A:

$$b(t) = \sum_{i=0}^m b_{m-i}^0 t^{m-i} + \sum_{i=1}^{m_1} b'_{m_1-i} t^{m_1-i} + \sum_{i=1}^{m_2} b''_{m_2-i} t^{m_2-i},$$

where:

$$\begin{aligned} b_{m-i}^0 &= \sum_{k=0}^{m-i} \frac{b_{1k} b_{2, m-i-k}}{c_{m-i}^k}, \\ b'_{m_1-i} &= \sum_{k=0}^{i-1} (m_1 - k) C_{m_1-k-1}^{m_1-i} b_{1, m_1-k} \int_0^\infty (-\theta)^{i-k-1} c_2(\theta) d\theta, \\ b''_{m_2-i} &= \sum_{k=0}^{i-1} (m_2 - k) C_{m_2-k-1}^{m_2-i} b_{2, m_2-k} \int_0^\infty (-\theta)^{i-k-1} c_1(\theta) d\theta, \end{aligned} \quad (67)$$

$$\begin{aligned} c(t) &= b_{10} c_2(t) + b_{20} c_1(t) - \sum_{i=0}^{m_1-1} (m_1 - i) b_{1, m_1-i} \int_i^\infty (t - \theta)^{m_1-i-1} c_2(\theta) d\theta - \\ &\quad - \sum_{i=0}^{m_2-1} (m_2 - i) b_{2, m_2-i} \int_i^\infty (t - \theta)^{m_2-i-1} c_1(\theta) d\theta + \\ &\quad + \sum_{k=0}^{n_1} [c_1(\theta_{1k}+) - c_1(\theta_{1k}-)] c_2(t - \theta_{1k}) + \sum_{k=1}^{n_1} \int_{\theta_{1,k-1}}^{\theta_{1k}} c_2(t - \theta) c'_1(\theta) d\theta. \end{aligned} \quad (68)$$

In order to find out the frequency function of the resultant system A, we assume that a sinusoidal signal  $x(t) = A \sin(\omega t + \varphi)$  is applied at the entry of the system. Using the complex representation of the sinusoidal values and taking into consideration the relation (14), in a permanent regime, the sinusoidal components of the exit signals from the systems  $A_1$  and  $A_2$  are written as follows:

$$Y_1(t) = F_1(j\omega)X(t),$$

$$Y(t) = F_2(j\omega)Y_1(t) = F_2(j\omega)F_1(j\omega)X(t),$$

resulting the frequency functions of the system A:

$$F(j\omega) = F_1(j\omega)F_2(j\omega),$$

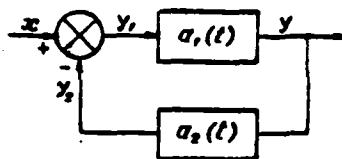
$$U(\omega) = U_1(\omega)U_2(\omega) - V_1(\omega)V_2(\omega),$$

$$V(\omega) = U_1(\omega)V_2(\omega) + V_1(\omega)U_2(\omega).$$

## 12. The connection with reaction

Let us assume that the system A, formed of the systems A<sub>1</sub> and A<sub>2</sub>, by a connection with reaction (fig. 7), is of EP type, and that a sinusoidal signal  $x(t) = A \sin(\omega t + \psi)$  is applied at the entry of the system.

Fig. 7. - Systems in connection with reaction



Using the complex representation of the sinusoidal values and taking into account the relation (14), in a permanent regime, we have

$$Y(t) = F(j\omega)X(t),$$

and from here

$$Y_2(t) = F_2(j\omega)Y(t) = F_2(j\omega)F(j\omega)X(t),$$

$$Y_1(t) = X(t) - Y_2(t) = [1 - F_2(j\omega)F(j\omega)]X(t),$$

$$Y(t) = F_1(j\omega)Y_1(t) = F_1(j\omega)[1 - F_2(j\omega)F(j\omega)]X(t).$$

From the two expressions of the sinusoidal component  $Y(t)$  of the exit signal from the system A, it results

$$F(j\omega) = \frac{F_1(j\omega)}{1 + F_1(j\omega)F_2(j\omega)},$$

$$\begin{aligned} U(\omega) &= \\ &= \frac{U_1(\omega)[1 + U_1(\omega)U_2(\omega) - V_1(\omega)V_2(\omega)] + V_1(\omega)[U_1(\omega)V_2(\omega) - V_1(\omega)U_2(\omega)]}{[1 + U_1(\omega)U_2(\omega) - V_1(\omega)V_2(\omega)]^2 + [U_1(\omega)V_2(\omega) - V_1(\omega)U_2(\omega)]^2}, \\ V(\omega) &= \quad (69) \\ &= \frac{-U_1(\omega)[U_1(\omega)V_2(\omega) + V_1(\omega)U_2(\omega)] + V_1(\omega)[1 + U_1(\omega)U_2(\omega) - V_1(\omega)V_2(\omega)]}{[1 + U_1(\omega)U_2(\omega) - V_1(\omega)V_2(\omega)]^2 + [U_1(\omega)V_2(\omega) - V_1(\omega)U_2(\omega)]^2}. \end{aligned}$$

In order that the resultant system A is of EP type, it is necessary that the real frequency functions  $U(\omega)$  and  $V(\omega)$  given by the relations (69) satisfy the necessary and sufficient conditions so that every one of them is able to characterize an EP type system, and the index functions of these systems have identical "exponential" components, that is

$$\int_0^{\infty} \frac{U_c(\omega)}{\omega} \sin \omega t \, d\omega = \int_0^{\infty} \frac{V_c(\omega)}{\omega} \cos \omega t \, d\omega.$$

It would be interesting to establish under which conditions this theorem also ensures the sufficiency; up to now, the author has not found a counter-example to confirm it.

#### OBSERVATIONS

a) If the resultant system A is stable, and the real frequency functions  $U(\omega)$  and  $V(\omega)$  given by the relations (69) satisfy the necessary and sufficient conditions so that every one of them is able to characterize an EP type system, then the system A is of EP type.

b) Let us assume that the system formed by connecting the systems  $A_1$  and  $A_2$  is described by the differential equations with delayed concentrations [9]. In conformity with the Nyquist criterion, for the resultant system A to be stable, it is necessary and sufficient that the hodograph of the function  $F_1(j\omega)F_2(j\omega)$  not intersect the real axis in the interval  $(-\infty, -1]$ .

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